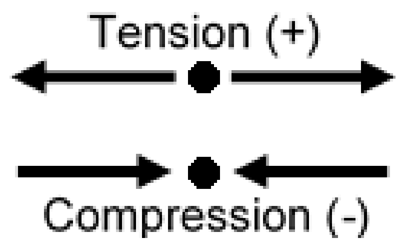
Trusses

A truss is a structure composed of members and joints that functions as a single object to counter any forces applied to it. Members are labeled by the joints at each of their ends. For example, if a triangular truss has three connected joints labeled A, B, and C, it would have the members AB, BC, and AC. Finding the amount of force each member experiences requires a two-step process. First, any unknown reaction forces supplied by the truss's supports must be solved by treating the truss as a rigid body with no internal members or features. Once all external forces are known, the internal members and joints can be examined using one of the two methods discussed in this handout: the method of joints or the method of sections. This handout will cover the details of the two different methods as well as how to set up and solve different problems with them.

You can navigate to specific sections of this handout by clicking the links below.

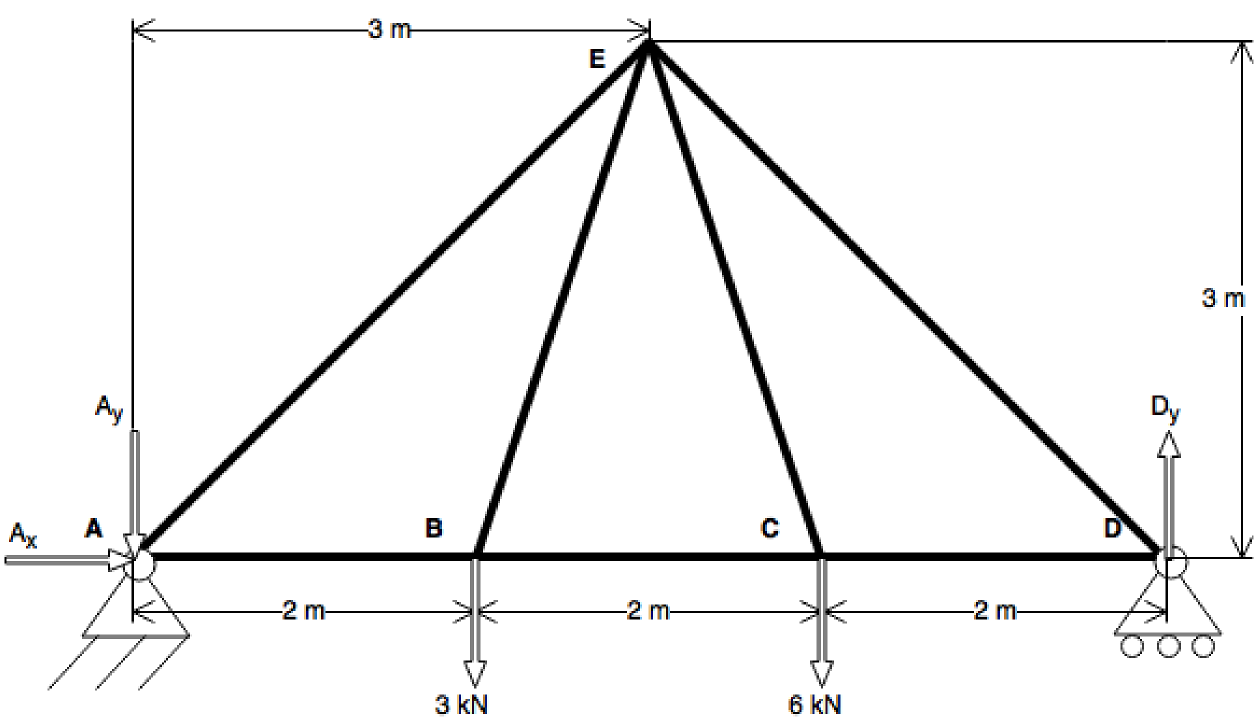
Tension and Compression

In any truss, members can experience one of two types of forces: tension or compression. Correctly identifying the type of force a member experiences is a crucial part of a solution, which is done by attaching a word or a sign to each force. Tension occurs when the member is pulled from both ends while compression occurs when it is pushed inward from each end, as illustrated in the figure to the right. If a member experiences neither a tensile or compressive force, then it is called a "zero-force member." When solving a problem, tension is always written as a positive value, and compression is always written as a negative value.



Solving for Reaction Forces

Consider the truss in the figure below, it will be used for all explanations throughout the handout. First, start by examining the forces on the entire structure. The two supports in this problem are a fixed pin on the left side and a roller on the right. Therefore, the unknown forces at joint A are A_x and A_Y, while the only unknown force at joint D is D_Y because rollers do not produce a reactionary force in the x direction.



Remember that a support describes what type of reaction forces occur at a specific point. While there are a number of different types of supports, the symbols that represent them are mostly the same across engineering textbooks.

The unknown reaction forces can be found as follows:

$$\Sigma F_x = \boldsymbol{A_x} = \mathbf{0}$$

$$\Sigma F_y = -A_y + D_y - 3kN \ - \ 6kN = 0$$

$$A_y = \ D_y - 9kN$$

$$\Sigma M_A = -(2\text{ m})(3\text{ kN}) - (4\text{ m})(6\text{ kN}) + (6\text{ m})D_y = 0$$

$$(6\,m)\ D_y = (6\ kN \cdot m) + (24\ kN \cdot m)$$

$$(6\,m)\ D_y = (30\ kN \cdot m)$$

$$\boldsymbol{D_y = 5\ kN}$$

The sum of the moments were calculated here because there were three unknown forces

$$A_y = \ D_y - 9kN$$

$$A_y = 5kN - 9kN$$

$$\boldsymbol{A_y = \ -4kN}$$

Substitute D_y into the equation previously identified from ΣF_y

$$\boldsymbol{D_y = 5\ kN}\ \mid\ \boldsymbol{A_y = -4\ kN}\ \mid\ \boldsymbol{A_x = 0\ kN}$$

Recall that the equations of equilibrium were established, assuming that A_y was directed downward. However, a negative result from the calculations means that this assumption was incorrect. Therefore, the force must be directed in the opposite direction (in this case, upwards), and it will be a positive value in future calculations. Now that the reaction forces have been identified, the internal forces can be analyzed using the method of joints or the method of sections.

Solving Trusses: Method of Joints

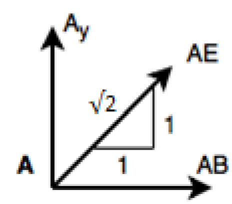
The method of joints evaluates the amount of force for every single member in the structure. In this method, free-body diagrams are drawn for each joint in the truss. For more on free-body diagrams see the Academic Center for Excellence's Free-Body Diagrams handout.

Each joint is in static equilibrium, so the forces are summed up in the x and y directions. Sometimes the moment about a joint must also be calculated in order to solve for all the unknown forces. Using the moment is a helpful tool when there are more than two unknown forces acting on the truss, but it may also be used when there are less.

Joint A:

$$\Sigma F_x = \frac{1}{\sqrt{2}} T_{AE} + T_{AB} = 0$$

$$\Sigma F_y = \frac{1}{\sqrt{2}} T_{AE} + 4kN = 0$$



After setting up the equations of equilibrium, find the unknown forces along the members by solving the system.

$$\boldsymbol{T_{AE} = -4\sqrt{2}\ kN\ (Compression)}$$

$$\boldsymbol{T_{AB} = 4\ kN\ (Tension)}$$

Joint B:

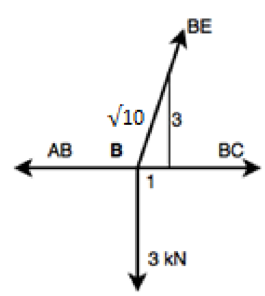
$$\Sigma F_x = -T_{AB} + T_{BC} + \frac{1}{\sqrt{10}} T_{BE} = 0$$

$$\Sigma F_y = \frac{3}{\sqrt{10}} T_{BE} - 3kN = 0$$

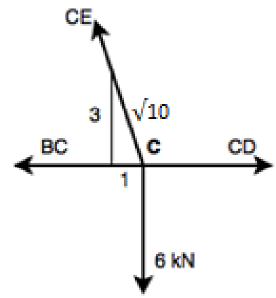
$$\boldsymbol{T_{BC} = 3\ kN\ (Tension)}$$

$$\boldsymbol{T_{BE} = \sqrt{10}\ kN\ (Tension)}$$

$\boldsymbol{T_{AB}}$ was already found in the previous free-body diagram for Joint A



<u>**Joint C**</u>:



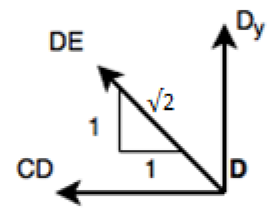
$$\Sigma F_x = -T_{BC} + T_{CD} - \frac{1}{\sqrt{10}} T_{CE} = 0$$

$$\Sigma F_y = \frac{3}{\sqrt{10}} T_{CE} - 6kN = 0$$

$$\boldsymbol{T_{CE} = 2\sqrt{10}\ (Tension)}$$

$$\boldsymbol{T_{CD} = 5\ kN\ (Tension)}$$

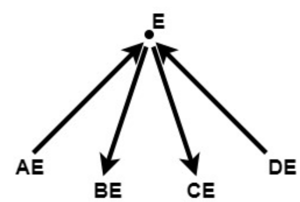
<u>**Joint D**</u>:



$$\Sigma F_x = -\frac{1}{\sqrt{2}} T_{DE} - T_{CD} = 0$$

$$\Sigma F_y = \frac{1}{\sqrt{2}} T_{DE} + 5kN = 0$$

$$\boldsymbol{T_{DE} = -5\sqrt{2}\ kN\ (Compression)}$$



Now all of the unknown forces have been solved. Notice that point E was not needed to solve for any forces. Because of this, it can be used as a means of checking the answers. By setting the equations of equilibrium for joint E and using values from the previous calculations, the answers will be confirmed as long as the equations cancel out to zero.

$$\Sigma F_x = \frac{1}{\sqrt{2}}\left(4\sqrt{2}\ kN\right) - \frac{1}{\sqrt{10}}\left(\sqrt{10}\ kN\right) + \frac{1}{\sqrt{10}}\left(2\sqrt{10}\ kN\right) - \frac{1}{\sqrt{2}}\left(5\sqrt{2}\right) = 0$$

$$4 - 1 + 2 - 5 = 0$$

$$\boldsymbol{0 = 0}$$

$$\Sigma F_y = \frac{1}{\sqrt{2}}\left(4\sqrt{2}\ kN\right) - \frac{3}{\sqrt{10}}\left(\sqrt{10}\ kN\right) - \frac{3}{\sqrt{10}}\left(2\sqrt{10}\ kN\right) + \frac{1}{\sqrt{2}}\left(5\sqrt{2}\right) = 0$$

$$4 - 3 - 6 + 5 = 0$$

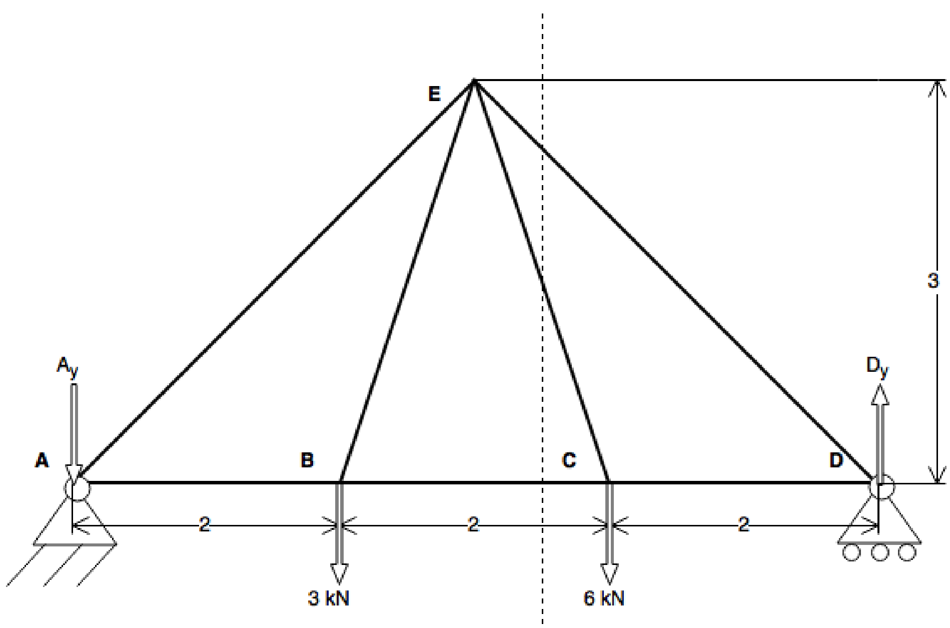
$$\boldsymbol{0 = 0}$$

Solving Trusses: Method of Sections

The method of sections is commonly used when the forces on only a few members need to be found. To do so, the truss is cut so that the member(s) can be more efficiently analyzed. When working with this method, a “cut” is an imaginary line drawn through the structure that isolates a section of the truss with the goal of simplifying the problem. It is important to remember that the cut may not intersect more than three members, or the problem becomes unsolvable. The cut members are converted into forces, which are then described in a free-body diagram that can be evaluated by using the equations of equilibrium.

Example

Suppose a problem asks for the force supplied by member CE in the following diagram. Begin with the whole structure, and make a cut that intersects the desired member, remembering to not cross more than three members. Based on those requirements, this example makes a cut at members BC, CE, and DE. The right portion of the cut will be used to solve the problem because it has fewer unknown forces at its joints than the other side.



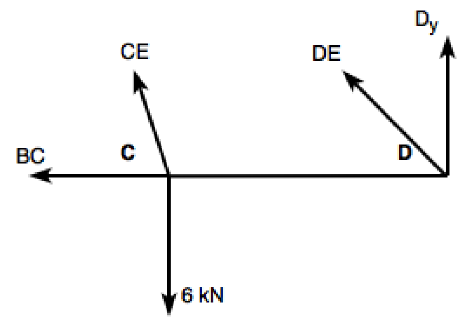
GERMANNA
ACADEMIC CENTER
FOR EXCELLENCE

Next, convert the members that were cut into forces, take the summations of forces with the remaining reaction forces, and apply the equations of equilibrium. This will result in the following free-body diagram and summations. Remember that all reaction forces are already solved for in the first section. Also, note that the new structure is analyzed as a free-body, so internal forces, such as F_{CD}, are not considered. Please refer to our Free-Body Diagrams handout for more information on working with free body diagrams.

$$\Sigma F_x = -T_{BC} - \frac{1}{\sqrt{10}}T_{CE} - \frac{1}{\sqrt{2}}T_{DE} = 0$$

$$\Sigma F_y = D_y + \frac{3}{\sqrt{10}}T_{CE} + \frac{1}{\sqrt{2}}T_{DE} - 6kN = 0$$

$$\Sigma M_D = (6\ kN)(2\ m) - \frac{3}{\sqrt{10}}(T_{CE})(2\ m) = 0$$



Of the three equations, ΣM_D is the most useful due to it only containing one variable (T_{CE}).

$$\Sigma M_D = 12\ kNm = \frac{6\text{ m}}{\sqrt{10}}T_{CE}$$

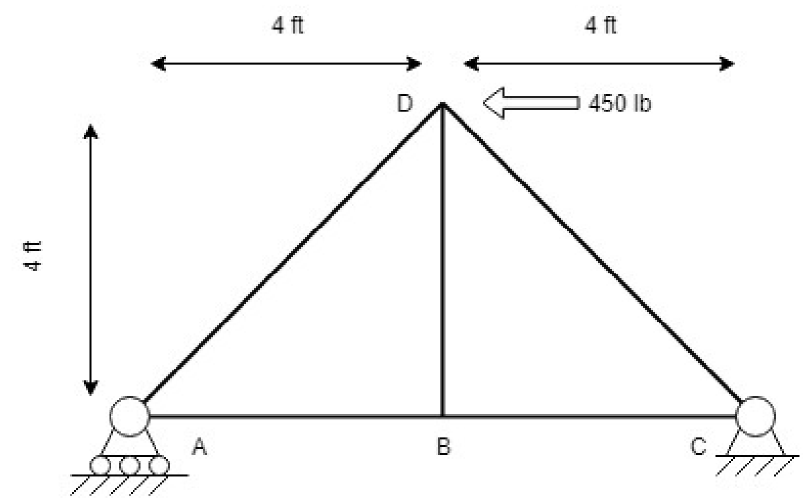
$$\boldsymbol{T_{CE} = 2\sqrt{10}\ kN}\ \textbf{tension}$$

This example demonstrates the efficiency of the method of sections when not all forces must be found because the problem was completed by using only one equation, whereas finding the same force using the method of joints would have taken far more equations and time to solve.

Practice problems for both methods can be found on the following pages, along with the answers to the problems at the end of this handout.

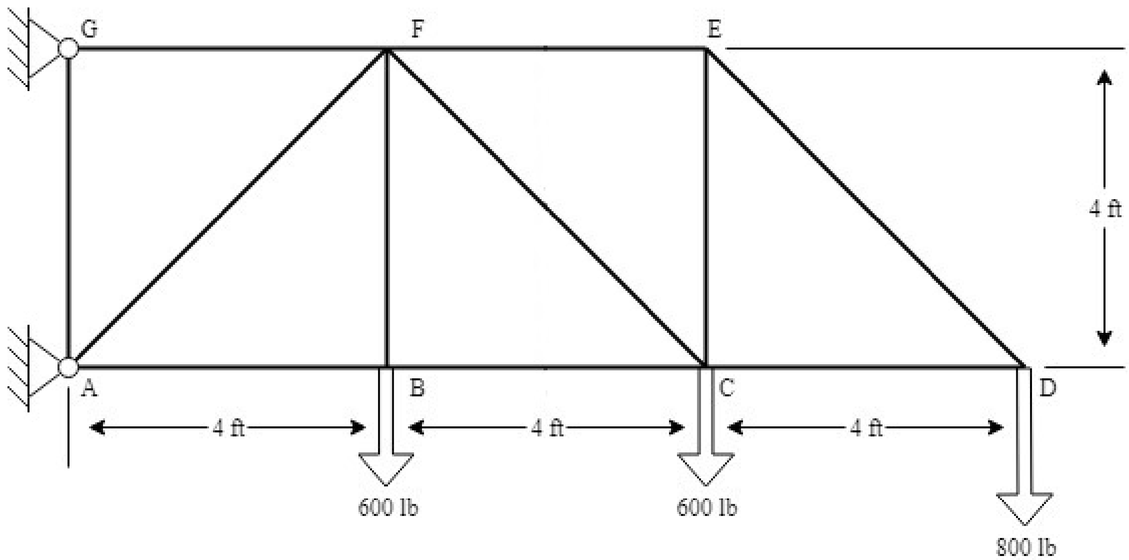
Practice Problems: Joints and Sections

Problem 1:



Find the forces in every member of the truss pictured above. State whether each member is in tension or compression (see page 10 for solution).

Problem 2:



For the truss pictured above, find forces BC, CF, and FE. State whether each member is in tension or compression (see page 10 for solution).

Solutions:

1) AD = 318.2 lb compression

   AB = 225 lb tension

   BC = 225 lb tension

   BD = 0 lb

   CD = 318.2 lb tension

2) FE = 800 lb tension

   CF = 1980 lb tension

   BC = 2200 lb compression