Moments and Torques

When a force is applied to an object, the object reacts in six possible ways. It can elongate, compress, translate (moves left, right, up, down, etc.), bend, twist or rotate. The study of moments and torques focuses on the bending, twisting or rotating of an object that has a force applied to it. More importantly, a moment is the quantity that describes the reaction of an object to a force where the object is not expected to rotate, such as a flag pole on the side of a building; whereas, torque is a quantity that describes the reaction of an object to a force where the object is expected to rotate, such as a wheel. The analysis of moments and torques begins with identifying a point (pivot) around which the object might (but not necessarily) rotate. In the study of mechanics, the tendency of an object to rotate, bend or twist is called the **moment of a force** about an axis (or point). Both moments and torques are vector quantities.

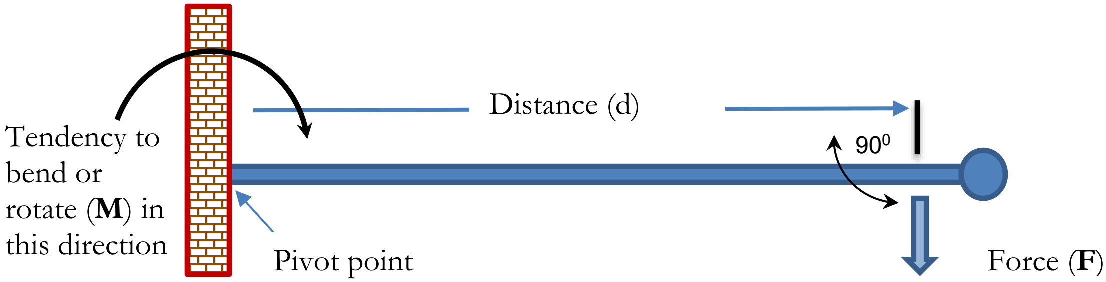
You can navigate to specific sections of this handout by clicking the links below.

Moments

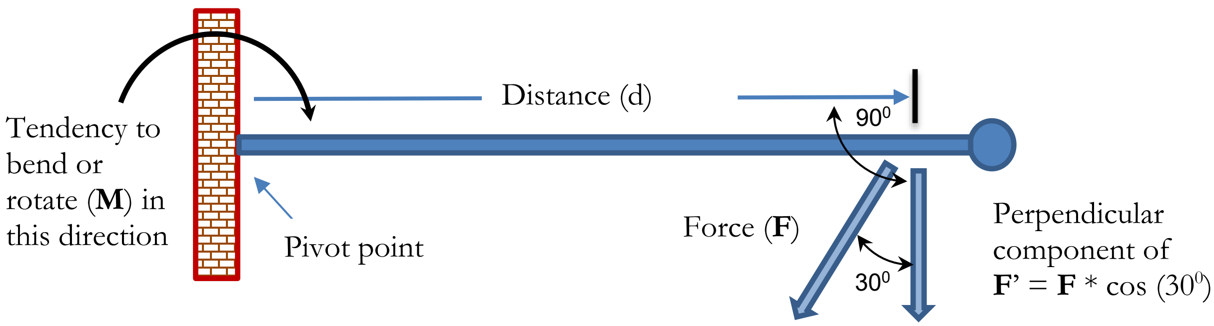
As shown in the following diagram of a flag pole mounted to a wall, a force applied at the end results in a rotating tendency of the pole (lever). This assumes that the pole does not bend. The magnitude of the moment is determined by the following:

$$M = F \cdot d$$

With the units of M being Newton•meters (Nm) or Pound Force•foot (lbf ft). Since the force applied is a vector (with magnitude and direction), and the distance (d) is a scalar, the resulting value (**M** or moment) is a **vector**. For now, we will consider only the magnitude of **M**. Note, vector quantities are bold.



The value of the force used when calculating the moment must be the force component that is **perpendicular** to the distance line drawn from the point of application of the force to the pivot point. If the force is applied at an angle, the component of the force that is perpendicular to the distance line must be found, and the component is then used to calculate the moment. The example below shows a situation where the force is applied to the pole at an angle.



The force used to calculate the moment in this case is F' = F * cos(30^0).

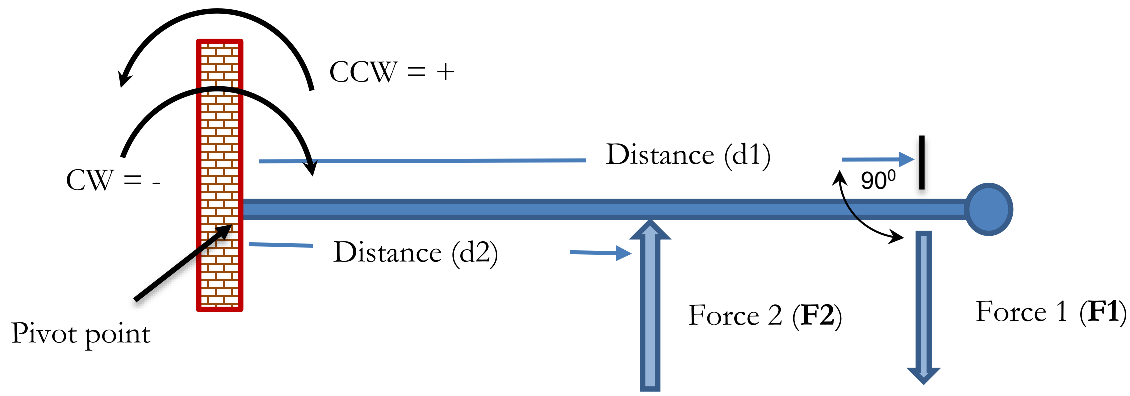
Moment as a Vector

We have only been considering the magnitude of the moment. The quantity moment is a vector that has magnitude and direction. The direction of the vector quantity is determined by using the right hand rule. The fingers of the right hand are oriented in the direction of possible rotation and the thumb determines the direction of the vector.

If you refer to our flag pole example, you will note that the tendency of the pole is to rotate clockwise. By using the right hand and aligning the fingers in the direction of rotation, the thumb will point into the page. Therefore the direction of the moment vector for the flagpole example is into the page. This is called the right-hand rule.

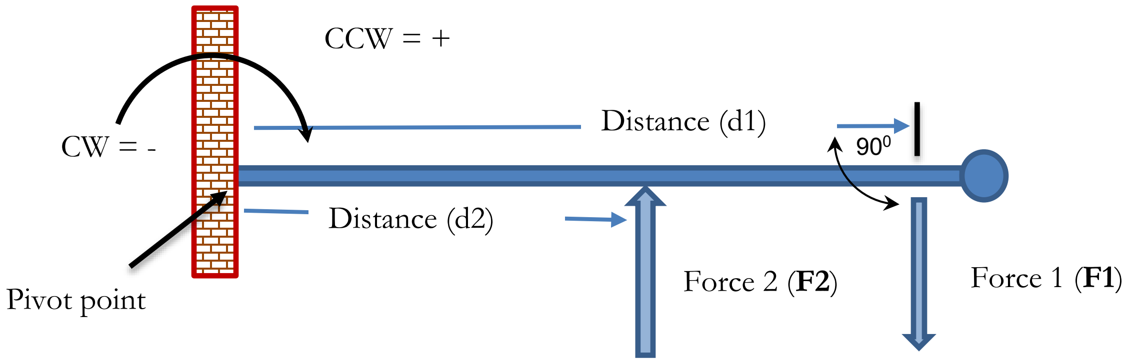
Sum of Moments

The total moment around a point is the sum of all moments around that point. In the case where multiple forces are being applied to a rigid body, the total moment can be calculated by simply adding the vector quantities of each **individual moment** created by each **individual force**. When summing all the moments applied to an object, the sign of each moment must be considered. A straightforward technique for adding moment vectors is to choose a direction of rotation as positive. For example, we can choose counter clockwise (CCW) rotation as positive, then any moment resulting in a tendency for the object to rotate in the CCW will have a positive sign. Likewise, any moment that results in a clockwise (CW) tendency of rotation will have a negative sign. The sum of the moments is then simply the algebraic sum of all the moments. The sign of the result then determines the ultimate direction of the tendency of the object to rotate based on the initial choice of sign and direction. Thus, using the right-hand rule, the actual direction of the resulting moment vector can be determined.



M1 = **F1** * d1
M2 = **F2** * d2
∑**M** = -**M1** + **M2**

Force **F2** will result in a moment in the CCW direction, so the moment has a positive value based on our previous choice of sign and direction. Force **F1** will result in a moment in the CW direction, so it has a negative value. If the resulting sum produces a total moment that has a positive value, then the object will have a tendency to rotate in the CCW direction. If the resulting sum produces a total moment that has a negative value, the object will have a tendency to rotate in the CW direction. In order for an object to remain stationary and therefore not rotate, the sum of the moments must equal zero. When referring to the diagram below, the wall must create a moment of equal magnitude and opposite direction to the moment created by the forces on the flag pole in order for the flag pole to remain stationary.



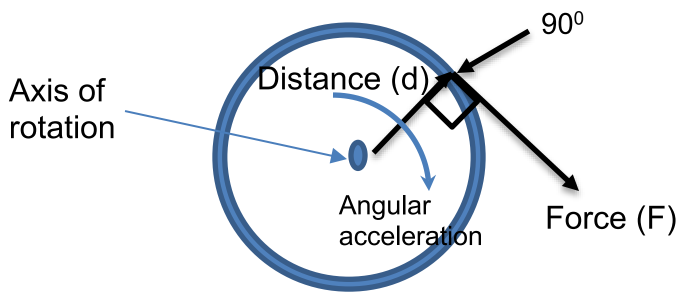
$$\mathbf{M1} = \mathbf{F1} * d1$$
$$\mathbf{M2} = \mathbf{F2} * d2$$
$$\sum \mathbf{M} = -\mathbf{M1} + \mathbf{M2}$$

Torque

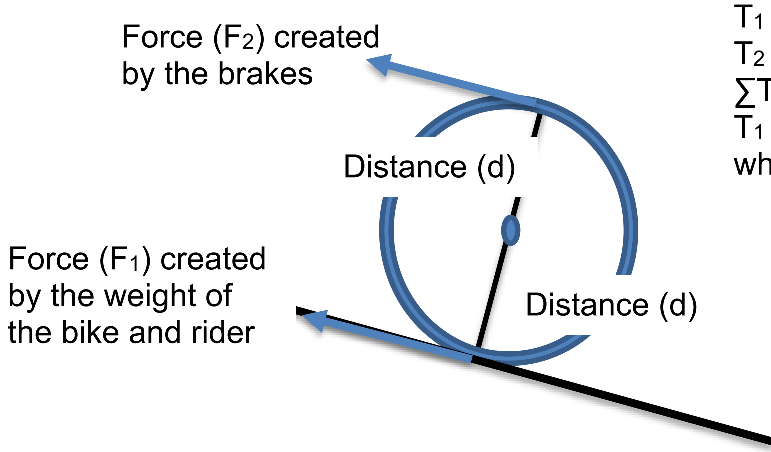
All of the rules, techniques and guidelines described for moments apply to the concept of torque; however, the object being analyzed is free to rotate unlike our flag pole example. Because the analysis of torque applies to objects that rotate (such as a wheel), applied forces that result in a torque will cause a rotating object to experience an angular acceleration, similar in concept to an object accelerating when a force is applied. The pivot point is typically the axis of rotation. The torque is then determine by:

$$T = F \bullet d$$

Torque has units of Newton•meter (Nm), or pound force•foot (lbf ft). As with moments, the direction of the torque vector is determined by using the right-hand rule, and the total torque is the sum of all the torques applied to an object. For example, a frictional force could be applied to the wheel in the opposite direction to the force (F) applied, thereby counteracting the torque generated by the force (F).



Unlike moments, when the sum of the clockwise and counterclockwise torques are equal (the sum of the torques = 0), the object may continue to rotate; however, it will not experience an angular acceleration. Consider the case of applying brakes to maintain a constant speed when riding a bike down a hill. When the torque created by the force of the brakes equals the torque created by the weight of the bike going downhill, the bike will not accelerating; however it will continue to move.



$T_1 = F_1 * d$
$T_2 = F_2 * d$
$\sum T = -T_1 + T_2$
T_1 is negative because it causes the wheel rotate in the CW direction