Free-Body Diagrams

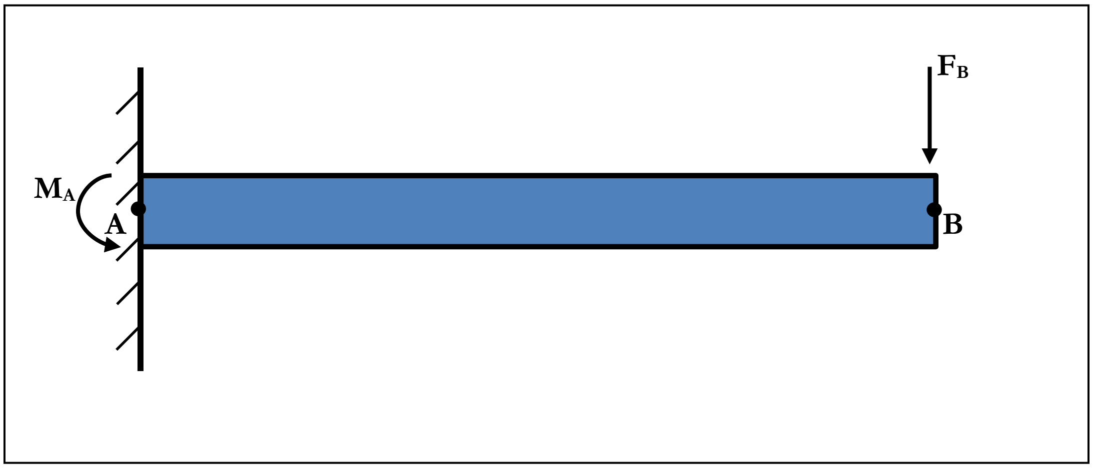
Introduction

A Free-Body Diagram is a basic two or three-dimensional representation of an object used to show all present forces and moments. The purpose of the diagram is to deconstruct or simplify a given problem by conveying only necessary information. Students may use this diagram as a reference for setting up calculations to find unknown variables such as force directions, force magnitudes, or moments. Free-Body Diagrams allow students to clearly visualize a problem in its entirety or closely analyze a portion of a more complex problem.

You can navigate to specific sections of this handout by clicking the links below.

Drawing a Free-Body Diagram

In a Free-Body Diagram, the object is represented by its simplest expression, usually a line, box, or dot. The force vectors acting upon the object are represented by straight arrows (→) while moments are represented by curved arrows (↻) around their respective axis as shown in the diagram below where a force acts at **B** and a moment around **A**. The force vectors indicate the magnitude and direction of each force that is acting upon the object. The direction is often indicated by degrees from the horizontal or vertical axis while the magnitude is indicated by units of force. In the case of an unknown magnitude or direction of a force, the unknown value must be labeled as such.



In addition, it is common to indicate various types of forces with letters and distinguish between common ones by using subscripts. In the example on page 3, weight and tension are represented by $\boldsymbol{W}$ and $\boldsymbol{T}$ respectively, and the force of friction and the normal force are represented by $\boldsymbol{F}_{\text{frict}}$ and $\boldsymbol{F}_{\text{norm}}$ respectively. There are no hard rules about how forces are labeled as long as the meaning is clear.

Free-Body Diagrams must also have a labeled coordinate system and include all given dimensions, such as length and angles. Generally, an xy-coordinate system will be used; however, when dealing with a problem in three-dimensional space, an xyz-coordinate system is required. Coordinate systems may be placed according to the student's discretion in order to simplify the solving process, so long as the student follows the right-hand rule (see Example Step 5 on page 5).

Types of Forces

In order to be able to identify and label forces for a Free-Body Diagram, students must recognize the various types of forces they will encounter and know how the forces interact with each other in order to calculate them.

Weight: Any object with a mass has a weight. It can be given in pounds or newtons (N). If the weight is not given, it can be calculated in newtons by multiplying the mass in kilograms with the Earth's gravitational constant ($9.8\ m/s^2$).

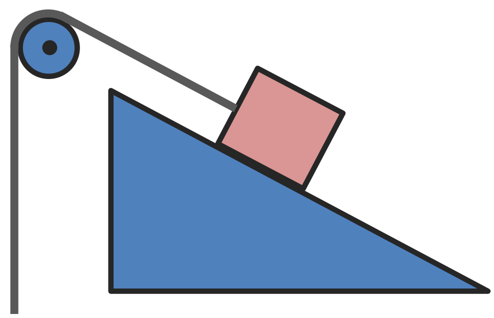
Normal Force: As explained by Newton's Third Law, every action has an equal and opposite reaction. Due to this law, any object that is not in free fall (i.e. an object that rests on a surface) has a normal force acting upon it perpendicular to the surface the object is resting on. Absent of any additional forces, the magnitude of the vertical component of the normal force is equal to the weight of the object.

Friction Force: Friction force resists movement and always acts in the opposite direction of movement or potential movement. It also acts parallel to the surface and is therefore perpendicular to the normal force. Friction force is equal in magnitude to the force providing the movement unless the opposing force exceeds the maximum friction force. The maximum friction force can be calculated by multiplying the normal force by the surface's coefficient of static friction.

Tension: Tension force is the pulling force exerted on an object by a rope, chain, cable, or other similar device. Tension force is continuous from one end to the other. Because tension force is continuous and rope is flexible, pulleys may be utilized to redirect the rope and by extension, the tension force.

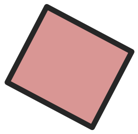
Applied Force: Applied force is any force that is applied by a person or by some other object.

Example:

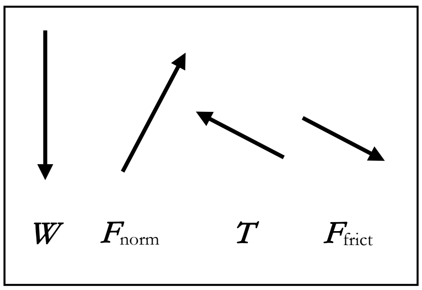


A 50 kg, stationary box must be pulled up a 30° incline by a pulley system. The coefficient of static friction between the box and the incline is 0.25. Assuming no friction in the pulley system, what force in newtons must be applied to the rope in order to move the box up the incline?

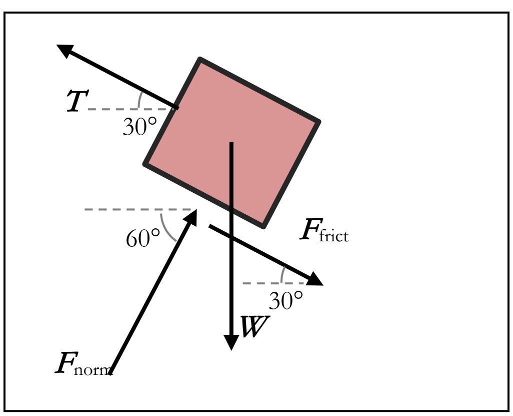
Setting up the Free-Body Diagram



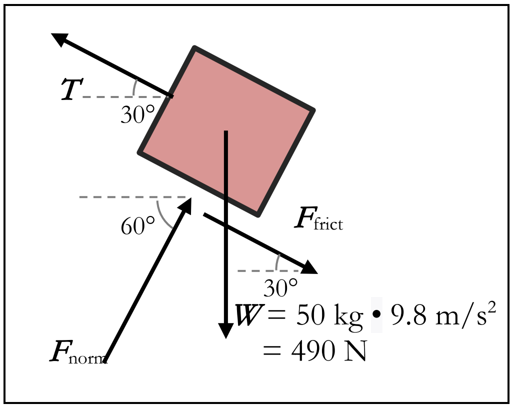
Step 1: Draw the object with no extraneous features



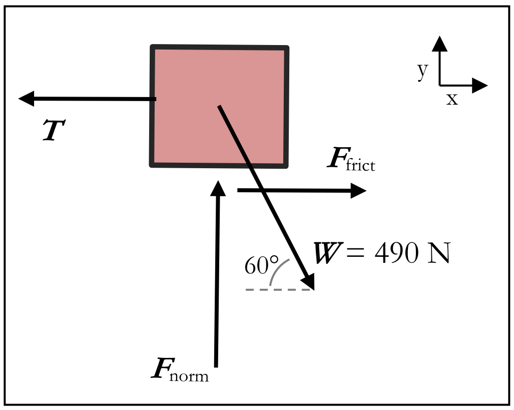
Step 2: Identify the forces present. The box has mass, so it also has **weight**, a force acting downward. Because the box is on a surface, there is a **normal force** acting perpendicular to the surface. Attached to the box, there is a rope with **tension** applied. This force will act in the direction of the rope. Since the rope is attached to the box, in order to move it up the incline, there will be a **frictional force** impeding movement. This force will act in the opposite direction, down the incline.



Step 3: Add the forces to the drawing of the object and label the directions of the forces in degrees from the vertical or horizontal axis as determined by the geometry in the example.



Step 4: Label all known values. At this point, all that is known is the weight, which is the mass (50 kg) multiplied by the gravitational constant (9.8 m/s^2). The Free-Body Diagram now contains all the given, important information.



Step 5: As a general rule, the Free-Body Diagram should be oriented, so the direction of movement is along one of the principle axes. In this example, the entire diagram can be reoriented by rotating it 30° counter-clockwise. This step results in the direction of movement occurring along the x-axis, and it results in three of the four forces also being oriented along the x or y axis.

Note: When dimensions are provided in the initial problem, it is essential to label all dimensions in the Free-Body Diagram.

Solving the Free-Body Diagram

In order to solve the problem, the force on the rope necessary to move the box up the incline must be found. This is the **tension** force. Finding this force requires a system of equations. Although there is currently one known variable, the **weight**, there are three unknown variables; therefore, three equations are required. These equations establish the relationship between each of the forces and are necessary in order to solve for each force.

In this system of equations, the first one establishes the relationship between the **normal** force and the **frictional** force. Because the box will not begin to move until the **tension** force overcomes the **frictional** force, the **max frictional** force is needed. The **max frictional** force is equal to the **normal** force multiplied by the surface's coefficient of static friction, which is 0.25, so the first equation will be written: $\boldsymbol{F}_{norm} \bullet 0.25 = \boldsymbol{F}_{frict}$

Because the question refers to the minimum force necessary to begin moving the object, the system is in static equilibrium just before the time of movement. Where a system in static equilibrium is concerned, there are two equations that are always applicable: $\sum \mathbf{F}_x = 0$ and $\sum \mathbf{F}_y = 0$, or in other words, the sum of all forces in the x and y direction must equal zero.

When all the forces in each direction are summed, it should be noted that any force that acts exclusively along either the x or y axis (in this case **tension**, **normal** force, and **frictional** force) will only be present in that axis's equation. However, the **weight** does not exclusively act along one axis, and therefore, must be broken down into its component parts. This is achieved by using trigonometric functions. In this case, because the **weight** acts at a 60° angle, the vertical component will be $\text{Sin}(60°) \bullet \boldsymbol{W}$, and the horizontal component will be $\text{Cos}(60°) \bullet \boldsymbol{W}$.

To set up the final two equations, the components of the various forces acting on each axis will be added together. Because of the direction of the positive x and y axes chosen in the previous step, forces on the x axis acting to the left will be negative, whereas forces acting to the right will be positive. Likewise, forces on the y axis acting downwards will be negative, and forces acting upwards will be positive. Looking at the Free-Body Diagram, three forces act in the x direction: the **tension** force, the **frictional** force, and the horizontal component of the **weight**. Two forces act in the y direction: the **normal** force and the vertical component of the **weight**. This leads to the final equations:

$\sum \mathbf{F}_x = \boldsymbol{F}_{frict} + \cos(60°) \bullet \boldsymbol{W} - \boldsymbol{T} = 0$ or $\boldsymbol{F}_{frict} + \cos(60°) \bullet \boldsymbol{W} = \boldsymbol{T}$

$\sum \mathbf{F}_y = \boldsymbol{F}_{norm} - \sin(60°) \bullet \boldsymbol{W} = 0$ or $\boldsymbol{F}_{norm} = \sin(60°) \bullet \boldsymbol{W}$

And also $\boldsymbol{F}_{norm} \bullet 0.25 = \boldsymbol{F}_{frict}$

Now that the system of equations is found, the unknown variables can be determined by inputting known information, namely that the **weight** is 490 N. Equations are then combined in order to solve for **tension**.

$\boldsymbol{F}_{norm} = \sin(60°) \bullet 490 \text{ N} = 424.4 \text{ N}$

$424.4 \text{ N} \bullet 0.25 = \boldsymbol{F}_{frict} = 106.1 \text{ N}$

$106.1 \text{ N} + \cos(60°) \bullet 490 = \boldsymbol{T} = 351.1 \text{ N}$

With **tension** found, the answer to the problem is at least 351.1 N of force must be applied to the rope in order to move the box up the incline.

When working with Free-Body Diagrams involving torque, please refer to the Academic Center for Excellence's handout, *Moments and Torques*.