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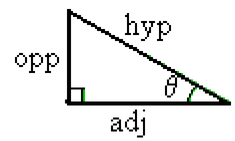
Common Trigonometric Angle Measurements



$\sin\theta = \frac{\text{opp}}{\text{hyp}}$ $\cos\theta = \frac{\text{adj}}{\text{hyp}}$ $\tan\theta = \frac{\text{opp}}{\text{adj}}$ $\csc\theta = \frac{\text{hyp}}{\text{opp}}$ $\sec\theta = \frac{\text{hyp}}{\text{adj}}$ $\cot\theta = \frac{\text{adj}}{\text{opp}}$

Hypotenuse = $\sqrt{\text{adjacent}^2 + \text{opposite}^2}$

Angle in Degrees	0°	30°	45°	60°	90°	120°	135°	150°	180°	210°	225°	240°	270°	300°	315°	330°	360°
Angle in Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
SIN	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$\frac{-1}{2}$	$\frac{-\sqrt{2}}{2}$	$\frac{-\sqrt{3}}{2}$	-1	$\frac{-\sqrt{3}}{2}$	$\frac{-\sqrt{2}}{2}$	$\frac{-1}{2}$	0
COS	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$\frac{-1}{2}$	$\frac{-\sqrt{2}}{2}$	$\frac{-\sqrt{3}}{2}$	-1	$\frac{-\sqrt{3}}{2}$	$\frac{-\sqrt{2}}{2}$	$\frac{-1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
TAN	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	ND	$-\sqrt{3}$	-1	$\frac{-\sqrt{3}}{3}$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	ND	$-\sqrt{3}$	-1	$\frac{-\sqrt{3}}{3}$	0
CSC	ND	2	$\sqrt{2}$	$\frac{2\sqrt{3}}{3}$	1	$\frac{2\sqrt{3}}{3}$	$\sqrt{2}$	2	ND	-2	$-\sqrt{2}$	$\frac{-2\sqrt{3}}{3}$	-1	$\frac{-2\sqrt{3}}{3}$	$-\sqrt{2}$	-2	ND
SEC	1	$\frac{2\sqrt{3}}{3}$	$\sqrt{2}$	2	ND	-2	$-\sqrt{2}$	$\frac{-2\sqrt{3}}{3}$	-1	$\frac{-2\sqrt{3}}{3}$	$-\sqrt{2}$	-2	ND	2	$\sqrt{2}$	$\frac{2\sqrt{3}}{3}$	1
COT	ND	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	$\frac{-\sqrt{3}}{3}$	-1	$-\sqrt{3}$	ND	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	$\frac{-\sqrt{3}}{3}$	-1	$-\sqrt{3}$	ND

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Angles	Q1	Q2	Q3	Q4
sin and csc	+	+	-	-
tan and cot	+	-	+	-
cos and sec	+	-	-	+

Reference Angles θ′

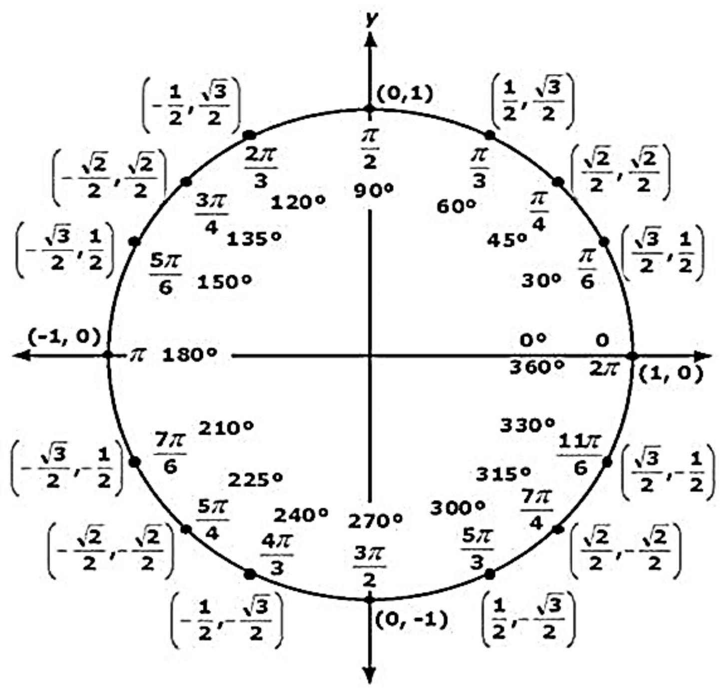
If…	$0º \leq \theta \leq 90$	then…	$\theta' = \theta$
	$90º < \theta \leq 180º$		$\theta' = 180º - \theta$
	$180º < \theta \leq 270º$		$\theta' = \theta - 180º$
	$270º < \theta \leq 360º$		$\theta' = 360º - \theta$

To convert from Radians to Degrees…

Multiply by $\frac{180°}{\pi\ \text{radians}}$

To convert from Degrees to Radians…

Multiply by $\frac{\pi\ \text{radians}}{180°}$



When using the unit circle:

Start at 0° and then…

Positive angles move counterclockwise.

Negative angles move clockwise.

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Fundamental Identities

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\cot\theta = \frac{\cos\theta}{\sin\theta}$ $\csc\theta = \frac{1}{\sin\theta}$ $\sec\theta = \frac{1}{\cos\theta}$ $\cot\theta = \frac{1}{\tan\theta}$

$\sin^2\theta + \cos^2\theta = 1$ $1 + \tan^2\theta = \sec^2\theta$ $1 + \cot^2\theta = \csc^2\theta$

Sum and Difference Identities

$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

Double-Angle Identities

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1$

$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$

Half-Angle Identities

$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$ $\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$ $\tan\frac{\alpha}{2} = \frac{\sin\alpha}{1+\cos\alpha} = \frac{1-\cos\alpha}{\sin\alpha} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$

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Even and Odd Identities

$\sin(-\theta) = -\sin\theta$ $\cos(-\theta) = \cos\theta$ $\tan(-\theta) = -\tan\theta$ $\csc(-\theta) = -\csc\theta$ $\sec(-\theta) = \sec\theta$ $\cot(-\theta) = -\cot\theta$

Power-Reducing Identities

$\sin^2\theta = \frac{1-\cos 2\theta}{2}$ $\cos^2\theta = \frac{1+\cos 2\theta}{2}$ $\tan^2\theta = \frac{1-\cos 2\theta}{1+\cos 2\theta}$

Product-to-Sum Identities

$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$ $\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha+\beta) + \cos(\alpha-\beta)]$

$\cos\alpha\sin\beta = \frac{1}{2}[\sin(\alpha+\beta) - \sin(\alpha-\beta)]$ $\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$

Sum-to-Product Identities

$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$ $\sin\alpha - \sin\beta = 2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$

$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$ $\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$

For sums of the form a sin x + b cos x…

$a\sin x + b\cos x = k\sin(x+\alpha)$

Where $k = \sqrt{a^2+b^2}$, $\sin\alpha = \frac{b}{\sqrt{a^2+b^2}}$, and $\cos\alpha = \frac{a}{\sqrt{a^2+b^2}}$

Arc Length Formula

$s = r\,\theta$ or $\theta = \frac{s}{r}$ radians

s = length of the arc
r = radius of the circle
θ = non-negative radian measure of the central angle

Converting from Degrees to Minutes to Seconds

1° (degree) = 60´ (minutes)

1´ (minute) = 60˝ (seconds)

1° (degree) = 3,600˝ (seconds)

Linear Speed (υ)

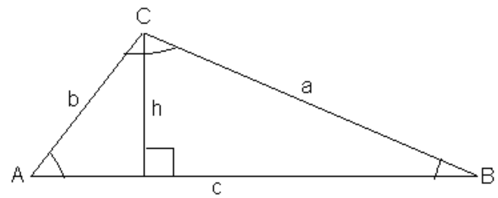
$\upsilon = \frac{s}{t}$ s = distance traveled
t = time

Angular Speed (ω)

$\omega = \frac{\theta}{t}$ θ = measure of angle in radians
t = time

Linear Speed – Angular Speed Relationship

$\upsilon = r\omega$ υ = linear speed
r = radius
ω = angular speed



Law of Sines: used to solve triangles when two angles and a side are given or when two sides and an opposite angle are given.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of Cosines: used to solve triangles when two sides and the included angle or three sides of the triangle are given.

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Area (K)

$$K = \frac{1}{2}bc\sin A = \frac{b^2 \sin C \sin A}{2\sin B}$$

$$K = \frac{1}{2}ab\sin C$$

$$K = \frac{1}{2}ac\sin B$$

Area (K) using Heron's Formula

$$K = \sqrt{s(s-a)(s-b)(s-c)}$$

$$s = \frac{1}{2}(a+b+c)$$